1.) You are given the following information about a stock:

- The growth rate of dividends over the first 10 years is g
- The value of the 23[rd] dividend is $15.00
- The growth rate of dividends from time 10 through time 20 is 2.0%
- The growth rate of dividends after year twenty is 1.5%
- The PV of the first 10 dividend payments is $100
- The effective annual yield is 4%

Calculate the price of the stock.

2.) You are given the following information about 4 bonds as of today, Dec 6[th], 2006:

Security	Years to Maturity	Annual Coupon	Yield-to-Maturity
A	1.0	0%	3.0%
B	2.0	0%	3.2%
C	3.0	6%	3.5%
D	4.0	5%	3.6%

Calculate the Spot Rate at time 4.

3.) You are given the following information with respect to a callable bond:

Time	Expected Cashflows at a 7% annual yield
1	8.00
2	7.90
3	107.80

Annual Yield	Bond Price
<=6.5%	104.33
7%	102.37
8%	99.76

The current yield is 7%.

Calculate the ratio of the effective duration to the effective convexity of this bond.

4.) A 2 year bond pays semi-annual coupons at an annual rate of 6% and is priced at par. Calculate the difference between the Macaulay duration and the Modified Duration.

5.) A common stock will pay a quarterly dividend of $0.50 in 3 months. Future quarterly dividends will grow at a rate of 3% per year. Five years from now the issuing company encounters financial difficulties and stops paying the scheduled dividends. Just less than 18 months after the financial difficulties began the company resumes paying the scheduled dividends. If the stock is sold at the end of 10 years for $25 what is the stock's price today at an annual effective yield of 9.75%?

6.) An insurance company has an obligation to pay 1 million dollars at the end of 10 years. It has a zero-coupon bond that matures for 413947.55 in 5 years, and it has a zero-coupon bond that matures for 864580.82 in 20 years. The current annual effective yield is 10%.

Is the company's liability portfolio fully immunized?

7.) A bank is required to pay 1,100 in one year. There are two investment options available with respect to how monies can be invested now in order to provide for the 1,100 payback:
(i) a non-interest bearing cash fund (ie i=0%), for which x will be invested, and
(ii) a two-year zero-coupon bond earning 10% per year, for which y will be invested.

Calculate the Modified Duration of this asset portfolio.

8.) You are the Chief Actuary at ManipuLife and you have been asked to determine the cost of a dedicated asset portfolio that will exactly match your company's liability cashflows. You are given the following information about the liabilities:

Time	1	2	3	4	5
Liability CF	64.50	464.50	852.50	1052.50	262.50

The following assets are available for purchase.

- 2 year bonds with annual coupons of 3% and price equal to $98.11 for $100 face amount
- 5 year bonds with annual coupons of 5% and price equal to $104.45 for $100 face amount
- 4 year bonds with annual coupons of 4%
- 3 year zero coupon bonds

These bonds can be purchased for any amount of face value. The annual effective yield on all 4 bonds is 4%.

9.) You are given the following with respect to a special five-year bond:

- annual coupons of $(2 + t)\%$ are paid at the end of each year
- par value of 1,000
- yield-to-maturity of 5.50%

Calculate the Convexity of this bond.

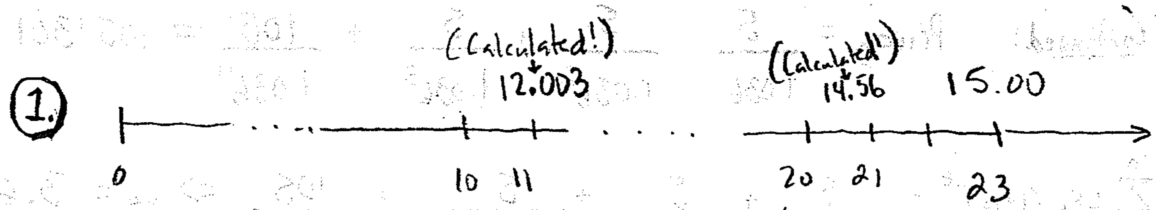
10.) You are given the following information:

- The spot rate at time 1 is 3%
- The spot rate at time 3 is 4.5%
- The spot rate at time 5 is 6%
- The price of a 4 year bond paying annual coupons at 5% is \$99.47 (face=\$100)
- The spot rate at time 2 equals the spot rate at time 4

Calculate the spot rate at time 4.

Test 4 - Solutions

1.



$Div_{21} = \dfrac{Div_{23}}{1.015^2} = 14.5599$ $\quad PV_{20}$ of future dividends $= \dfrac{Div_{21}}{r-g} = 582.4$

$$Div_{11} = \frac{Div_{23}}{(1.015)^3 \cdot (1.02)^9} = 12.003$$

$$PV_{10} \text{ of } Div_{11} \text{ to } Div_{20} = \frac{Div_{11}}{(.04-.02)} - \frac{Div_{11} \cdot (1.02)^{10}}{(.04-.02)} \cdot \left(\frac{1}{1.04}\right)^{10} =$$

$$Div_{11} \cdot \ddot{a}_{\overline{10}|\frac{1+r}{1+k}-1} \cdot \left[\frac{1}{1.04}\right] = 105.9215$$

$$\therefore \text{Price} = 100 + PV_{10} \cdot v^{10} + PV_{20} \cdot v^{20} = 100 + 71.557 + 265.80 =$$

$= \$\underline{437.36}$

2.

$$\text{Price}_A = \frac{1}{(1.03)} = .970874 = \frac{1}{(1+s_1)^1} \Rightarrow s_1 = .03 \text{ or } 3\%$$

$$\text{Price}_B = \frac{1}{(1.032)^2} = .938946 = \frac{1}{(1+s_2)^2} \Rightarrow s_2 = .032 \text{ or } 3.2\%$$

$$\text{Price}_C = \frac{6}{1.035^1} + \frac{6}{1.035^2} + \frac{106}{1.035^3} = 107.0041 = \frac{6}{(1+s_1)^1} + \frac{6}{(1+s_2)^2} + \frac{106}{(1+s_3)^3} =$$

$$\frac{6}{1.03^1} + \frac{6}{1.032^2} + \frac{106}{(1+s_3)^3} \Rightarrow s_3 = 3.52193\ \%$$

(2) (Continued) $\text{Price}_D = \frac{5}{1.036} \; \frac{5}{1.036^2} + \frac{5}{1.036^3} + \frac{105}{1.036^4} = 105.1301 =$

$$\sum_{t=0}^{n} CF_t \cdot (1+s_t)^{-t} = \frac{5}{(1.03)^1} + \frac{5}{(1.032)^2} + \frac{5}{(1.0352193)^3} + \frac{105}{(1+s_4)^4} \Rightarrow \underline{\underline{s_4 = 3.6212\%}}$$

(3.) Effective Duration $(\text{EffD}) = \frac{P_- - P_+}{P_0 (2\Delta y)}$ let $\Delta y = .01$

$P_- = 104.33$
$P_+ = 99.76$
$P_0 = 102.37$

$$\Rightarrow \text{EffD} = \frac{104.33 - 99.76}{102.37(2 \cdot .01)} = 2.2321$$

Effective Convexity $(\text{EffC}) = \frac{P_- + P_+ - 2P_0}{(\Delta y)^2 P_0} = \frac{104.33 + 99.76 - (2)(102.37)}{(.01)^2 \cdot (102.37)}$

$$= \frac{-.65}{.010237} = -63.4952$$

$$\text{ratio} = \frac{\text{EffD}}{\text{EffC}} = \frac{2.2321}{-63.4952} = \boxed{-.03515}$$

(4.) MacD can be solved two ways:

(1) $\text{MacD} = \ddot{a}^{(m)}_{\overline{2}|} = \frac{1 - v^2}{d^{(2)}}$ $\qquad d^{(2)} = 2\left[1 - \left(1 + \frac{i^{(2)}}{2}\right)^{-1}\right] = 5.82524\%$

$= \frac{1 - (.9426)^2}{.0582524} = 1.9143$ $\qquad v^2 = \left(1 + \frac{i^{(2)}}{2}\right)^{-2} = 1.03^{-2} = .9426$

(2) $\text{MacD} = \frac{\sum t \cdot CF_t \cdot (1 + y/m)^{-mt}}{P} = \frac{\sum t \cdot CF_t \cdot v_i^t}{P}$ let face equal \$100, Price = \$100 since bond priced at par.

$$\Rightarrow \frac{(.5) \cdot 3 \cdot (1.0609)^{.5} + (1)(3)(1.0609)^1 + (1.5)(3)(1.0609)^{1.5} + (2)(103)(1.0609)^2}{100} = 1.9143$$

(4) <u>Continued</u>

$$\text{Modified Duration} = \frac{\sum t \cdot CF_t \cdot (1+y/m)^{-mt}}{P \cdot (1+y/m)} = \frac{\sum t \cdot CF_t \cdot (1+y/m)^{-mt-1}}{P} = \frac{MacD}{(1+y/m)}$$

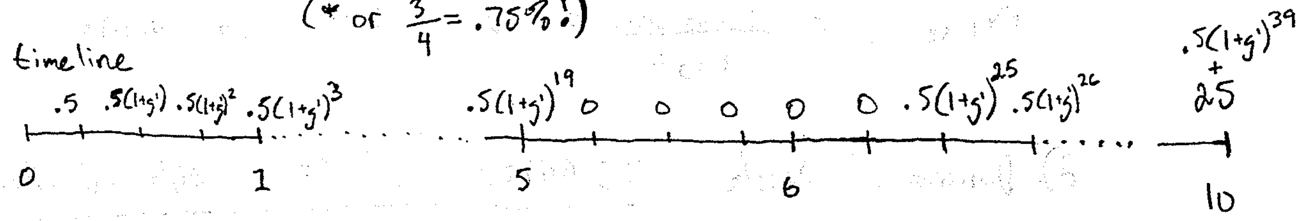
$\therefore$ if $MacD = 1.9143$, then $ModD = \dfrac{1.9143}{\left(1+\frac{.06}{2}\right)} = 1.8585$

$\Rightarrow$ difference $= MacD - ModD = 1.9143 - 1.8585 = \boxed{.05576}$

(*Note to markers, students could have solved for ModD first and then MacD using the formula $MacD = ModD \cdot (1+\frac{y}{m})$)

(5) $g' = (1+g)^{1/4} - 1 = (1.03)^{1/4} - 1 = .00741707$ (* or $\frac{3}{4} = .75\%$!) $\quad r' = (1+r)^{1/4} - 1 = 1.0975^{1/4} - 1 = .0235313$

timeline



$$\text{Price} = (.5)\left[\ddot{a}_{\overline{20}|\frac{1+r'}{1+g'}-1}\right] \cdot \frac{1}{(1+r')} + .5(1+g')^{25} \cdot \left[\ddot{a}_{\overline{15}|\frac{1+r'}{1+g'}-1}\right] \cdot \frac{1}{(1+r')^{26}} +$$

$$+\ 25 \cdot \frac{1}{(1+r')^{40}} = (.5)\left[17.2733\right] \cdot (.9770) + (.5)(1.20291)\left[13.45454\right] \cdot (.54622)$$

$$+\ 25 \cdot (.39442) = \underline{\$22.72}$$ * see note below

OR $$\text{Price} = (.5)\left[\frac{1}{r'-g'} - \frac{1 \cdot (1+g')^{20}}{r'-g'} \cdot \frac{1}{(1+r')^{20}}\right] + .5\left[\frac{1 \cdot (1+g')^{25}}{r'-g'} - \frac{1 \cdot (1+g')^{40}}{r'-g'} \cdot \frac{1}{(1+r')^{15}}\right] \cdot \frac{1}{(1+r')^{25}}$$

$$+\ 25 \cdot \frac{1}{(1+r')^{40}} = .5\left[16.87614\right] + .5\left[15.81249\right] \cdot (.55908) + 9.8604 = \underline{\underline{\$22.72}}$$ * see note below

Note to Markers will also accept $22.36 which is the excluding dividend 20!

(6.) For the portfolio to be immunized the following 3 conditions must be met:

1) PV assets = PV liability

2) Duration of assets = Duration of Liability

3) $(T-q) < T < (T+r)$

Need to check that all three are true...

1) $PV_{assets} = \frac{413947.55}{1.10^5} + \frac{864580.82}{1.10^{20}} = 385\,543.29$

$PV_{liability} = \frac{1000\,000}{1.10^{10}} = 385\,543.29$ so condition 1 is met ✓

2) Duration of Assets $= \frac{413\,947.55(1.10)^{-5}(5) + 864\,580.82(1.10)^{-20}(20)}{385\,543.29}$

$= 10$

Duration of Liability $= \frac{\sum t \cdot CF_t \cdot v_i^t}{Price} = \frac{(10)(1000\,000)\frac{1}{1.10^{10}}}{385\,543.29} = 10$

(OR Duration of Liabs = 10 since only 1 CF!)

Since 10 = 10 condition #2 is met ✓

3) • Asset 1 matures at time 5.
• Asset 2 matures at time 20.
• The liability matures at time 10.

∴ Since $5 < 10 < 20$ condition #3 is met.

$\boxed{\text{∴ Since all 3 conditions are met the portfolio is fully immunized!}}$

(7.) Duration of Portfolio $= \frac{P_1}{MV} \cdot D_1 + \frac{P_2}{MV} \cdot D_2$

$P_1 = x$ (since $i = 0\%$) $\quad P_2 = y \quad MV = P_1 + P_2 = x + y$

$$D_1 = \text{Mod}\,D_1 = \frac{\sum t \cdot CF_t \cdot v_i^t}{P_1 \cdot (1+i)} = \frac{(0)(x) \cdot v_i^0}{x(1+i)} = 0$$

$$D_2 = \text{Mac}\,D_2 = \frac{\sum t \cdot CF_t \cdot v_i^t}{P_2 \cdot (1+i)} = \frac{(2)(y(1.1)^2) \cdot \frac{1}{1+i^2}}{y \cdot (1+i)} = \frac{2 \cdot (1.1)^2}{(1+i)^3}$$

$$\text{Duration of Portfolio} = \frac{x \cdot 0}{x+y} + \frac{y \cdot \frac{2 \cdot (1.1)^2}{(1+i)^3}}{x+y} = \frac{2y \cdot (1.1)^2}{(x+y) \cdot (1+i)^3}$$

*Note to markers if the students get to here award 4 points.

$i = 10\%$ since zero-coupon bond earns 10% $\therefore$ Duration of Portfolio $= \frac{2y(1.1)^2}{(x+y)(1.1)^3} = \boxed{\frac{1.8182\,y}{(x+y)}}$

(8.)

	time	1	2	3	4	5
	Liability CF's	64.50	464.50	852.50	1052.50	262.50
asset 1 = 5 yr bond	Asset$_1$ CF's	12.50	12.50	12.50	12.50	262.50
	Remaining CF's	52	452	840	1040	0
asset 2 = 4 yr bond	Asset$_2$ CF's	40	40	40	1040	
	Remaining CF's	12	412	800	0	
asset 3 = 3 yr zero coupon bond	Asset$_3$ CF's	0	0	800		
	Remaining CF's	12	412			
asset 4 = 2 yr bond	Asset$_4$ CF's	12	412			

$$\text{Cost} = (2.5)(104.45) + (10)\cdot(100) + \frac{800}{1.04^3} + (4)(98.11) = \$\underline{\underline{2364.76}}$$

(note to markers, since all bonds yield 4%, shortcut is acceptable => Price = PV of liability cashflows at 4%!)

(9) coupons = 1000 (2+t)% $\Rightarrow$ Price = $\frac{30}{1.055} + \frac{40}{1.055^2} + \frac{50}{1.055^3} + \frac{60}{1.055^4} + \frac{1070}{1.055^5}$

Price = 974.08

$$\text{Convexity} = \frac{\sum t(t+1)\,CF_t\,(1+y)^{-t-2}}{\text{Price}} = \left[\frac{(1)(2)(30)}{(1.055)^3} + \frac{(2)(3)(40)}{1.055^4} + \frac{(3)(4)\cdot 50}{1.055^5} + \right.$$

$$\left.\frac{(4)(5)(60)}{1.055^6} + \frac{(5)(6)(1070)}{1.055^7}\right] \div 974.08 = \boxed{24.27}$$

(10) 4-yr bond $\frac{5}{1.03} + \frac{5}{(1+s_2)^2} + \frac{5}{1.045^3} + \frac{105}{(1+s_4)^4} = 99.47$

$\frac{5}{(1+s_2)^2} + \frac{105}{(1+s_4)^4} = 90.2341$, since $s_2 = s_4$

$\frac{5}{(1+s_2)^2} + \frac{105}{(1+s_2)^4} = 90.2341$ let $x = \frac{1}{(1+s_2)^2}$, then $x^2 = \frac{1}{(1+s_2)^2}$

using quadratic $105x^2 + 5x - 90.2341$ $x = .9035198$

$\Rightarrow .9035198 = \frac{1}{(1+s_2)^2}$ $s_2 = \underline{5.204}\%$ $= s_4$